

# Optical transitions and Rabi oscillations in waveguide arrays

K. G. Makris,<sup>1\*</sup> D. N. Christodoulides,<sup>1</sup> O. Peleg,<sup>2</sup> M. Segev,<sup>2</sup> and D. Kip<sup>3</sup>

<sup>1</sup>College of Optics and Photonics, CREOL & FPCE, University of Central Florida, 4000 Central Florida Blvd., Orlando Florida 32816, USA

<sup>2</sup>Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel

<sup>3</sup>Institute of Physics and Physical Technologies, Clausthal University of Technology, 38678 Clausthal-Zellerfeld, Germany

\*Corresponding author: [kmakris@creol.ucf.edu](mailto:kmakris@creol.ucf.edu)

**Abstract:** It is theoretically demonstrated that Rabi interband oscillations are possible in waveguide arrays. Such transitions can take place in optical lattices when the unit-cell is periodically modulated along the propagation direction. Under phase-matching conditions, direct power transfer between two Floquet-Bloch modes can occur. In the nonlinear domain, periodic oscillations between two different lattice solitons are also possible.

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**OCIS codes:** (130.2790) Guided Waves; (230.7370) Waveguides; (190.3270) Kerr effect.

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## 1. Introduction

It is a well known fact that when an electron “drops” from the conduction band to the valence band, the excess energy is converted into electromagnetic radiation (photons) or into lattice vibrations (phonons). This emission can take place spontaneously from zero-point quantum fluctuations or it can be induced through stimulated emission. On the other hand, light can also be absorbed in crystals, resulting in an electronic transition to a higher energy state (conduction band). What actually facilitates this process is in fact a periodic time perturbation added to the electron Hamiltonian due to the presence of an external optical field. In the absence of any phonon or defect interactions, such transitions obey selection rules and as a result they can only take place if the Bloch momentum is conserved (direct transitions). In addition, the energy must also be conserved. Therefore, the period of the harmonic perturbation should be equal to the energy difference between the two levels. Rabi oscillations in the framework of atom optics have been considered at both the theoretical [1] and experimental front [2].

In this context it is natural to ask if these same processes can also be observed in the optical domain, in particular in waveguide lattices. Already, there have been suggestions of observing such transitions in photonic crystal systems using either  $\chi^{(2)}$  nonlinear or ultrasonic time/space harmonic perturbations [3, 4]. However, thus far such all-optical transitions have never been observed experimentally.

In this paper, we show that optical Rabi inter-band transitions and nonlinear power exchange between two different gap solitons are possible in periodically modulated array structures (inset of Fig.1) [5, 6]. These transitions can occur among bands or gaps in the Bloch momentum  $k$ -space, as shown in Fig.1. Note that the normalized value of the propagation constant in Fig. 1 can be either positive or negative, when compared to the propagation constant of the background refractive index of the periodic potential.

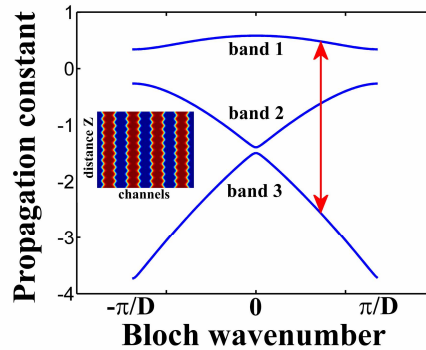


Fig. 1. Allowed all-optical direct transition between the first and third band. The inset depicts a top view of a periodically modulated optical lattice.

The array is modulated in the propagation direction  $z$  with a period  $\Lambda$ , in such a way that the “energy difference”  $2\pi/\Lambda$  spans the “energy/eigenvector”-difference between the allowed bands. This of course occurs provided that the transition selection rules are respected. Fig.1 shows such a possible direct transition between the first and the third band of this structure. In the case shown, this transition is allowed because the Floquet-Bloch (FB) modes involved possess the same parity. We note that these transitions occur as a result of parametric mixing and thus are different from the recently reported Zener tunneling [7, 8], as well as, from the Bragg-resonance induced transitions [9] and spatial four-wave-mixing effects [10].

## 2. Coupled mode theory of FB mode transitions

In order to understand the dynamics of inter-band optical transitions we derive the corresponding coupled-mode equations. This is done by using time-dependent perturbation techniques in the paraxial equation of diffraction. In this case, wave propagation in a transversely-periodic potential (unperturbed in  $z$ ) is governed by the following normalized equation  $iU_z + U_{xx} + V_0 U = 0$ , where  $U$  represents the optical field,  $z$  is the propagation distance,  $x$  the transverse coordinate, and  $V_0(x)$  is the periodic index potential with spatial period  $D$ ,  $V_0(x) = V_0(x + D)$ . The general solution of this problem can be expressed as a linear superposition of FB modes  $\phi_{k_n}(x)$ , where  $k$  denotes the Bloch wave vector and  $n$  the band-index number. In general,  $U$  can be expressed as a superposition

$$U(x, z) = \sum_{n=1}^{+\infty} \int_{-\pi/D}^{\pi/D} c_n(k) \phi_{k_n}(x) \exp[i\beta_n(k)z] dk, \quad \text{where } c_n(k) \text{ represent FB mode}$$

occupancy coefficients and  $\beta_{k_n}$  is the corresponding propagation constant. Thus the unperturbed eigenvalue problem is given by  $-\beta_n(k)\phi_{k_n} + \phi_{k_n}'' + V_0(x)\phi_{k_n} = 0$ . If on the other hand, the lattice is perturbed by a  $z$ -dependent periodic potential, then the system obeys:

$$i \frac{\partial U}{\partial z} + \frac{\partial^2 U}{\partial x^2} + V(x, z)U = 0 \quad (1),$$

where  $V(x, z)$  is a lattice potential periodically modulated along the  $z$ -direction. The perturbed  $z$ -dependent potential  $V(x, z)$  is written as  $V(x, z) = V_0(x) + \varepsilon \cdot V'(x, z)$ , where  $\varepsilon \ll 1$  and  $V'(x, z)$  is a weak periodic modulation. Note that  $V'(x, z)$  has the same periodicity as the original lattice  $V_0(x)$ , e.g.  $V'(x, z) = V'(x + D, z)$ . By writing

$$U(x, z) = \sum_{n=1}^{+\infty} \int_{-\pi/D}^{\pi/D} c_n(k, z) \phi_{k_n}(x) \exp[i\beta_n(k)z] dk, \quad \text{where the occupancy coefficients}$$

$c_n(k, z)$  are now  $z$ -dependent, we find

$$\sum_{n=1}^{+\infty} \int_{-\pi/D}^{+\pi/D} \left\{ i \dot{c}_n \langle \phi_{k'_m}, \phi_{k_n} \rangle \exp[i\beta_n(k)z] + \varepsilon \langle \phi_{k'_m}, V'(x, z) \phi_{k_n} \rangle c_n \exp[i\beta_n(k)z] \right\} dk = 0,$$

where  $\langle f, g \rangle \equiv \int_{-\infty}^{+\infty} f^*(x) g(x) dx$ . Evidently, the coupling strength between FB modes

depends on the transition matrix element  $\langle \phi_{k'_m}, V'(x, z) \phi_{k_n} \rangle$ . Given that both  $\phi_{k_n}(x)$  and  $V'(x, z)$  have the same period  $D$ , the latter inner product can then be reduced to

$$\langle \phi_{k'_m}, V'(x, z) \phi_{k_n} \rangle = \left( \int_{cell} \phi_{k'_m}^* V' \phi_{k_n} dx \right) \delta(k - k'). \quad \text{This last result indicates that only direct}$$

transitions are allowed, i.e., the transverse momentum  $k$  must be conserved in order for a transition to occur. By using the orthogonality condition  $\langle \phi_{k'_m}, \phi_{k_n} \rangle = \delta_{n,m} \delta(k - k')$  we can then derive the coupled mode equations describing the dynamics of Rabi-like oscillations. In this case, the FB mode occupancy coefficients associated with inter-band direct transitions,

satisfy the following equation,  $i \dot{c}_m + \sum_{n=1}^{+\infty} [A_{nm} c_n \exp(i\Delta\beta_{nm} z)] = 0$ , where  $\Delta\beta_{nm} = \beta_n(k) - \beta_m(k)$  and  $A_{nm} = \varepsilon \int_{cell} \phi_{km}^*(x) V'(x, z) \phi_{kn}(x) dx$  represents the overlap integral. By expressing the periodic perturbation in a separable form  $V'(x, z) = V'(x) \cos\left(\frac{2\pi}{\Lambda} z\right)$ , and dropping the resulting highly oscillating terms, one can derive the coupled mode equations that describe FB mode transitions from band  $n$  to band  $m$  and vice versa. These are:

$$\begin{aligned} i \frac{dc_n}{dz} + M_{mn} c_m \exp[-i(\Delta\beta_{nm} - 2\pi/\Lambda)z] &= 0 \\ i \frac{dc_m}{dz} + M_{nm} c_n \exp[i(\Delta\beta_{nm} - 2\pi/\Lambda)z] &= 0 \end{aligned} \quad (2)$$

where  $M_{nm} = \frac{\varepsilon}{2} \int_{cell} \phi_{km}^* V'(x) \phi_{kn} dx$ . From Eq. (2) it can be directly shown that the transition  $m \leftrightarrow n$  between two different FB modes is optimum under the phase-matching condition  $\Delta\beta = 2\pi/\Lambda$ . In this case, complete power transfer will occur at a coupling distance  $L = \pi / (2|M_{nm}|)$ .

### 3. Selection rule diagram and examples of FB mode transitions

As an example, we consider optical transitions in periodically modulated AlGaAs semiconductor lattices (waveguide arrays) when  $\Delta\beta = 2\pi/\Lambda$ . More specifically, these processes can be realized in systems with periodicity  $\Lambda \sim 0.15$  mm. This facilitates several cycles of oscillation in experimentally-realizable samples that can be 1-3 cm long. The period in this array is  $D = 8 \mu\text{m}$  and the waveguide width varies periodically ( $w = w_0 [1 + \varepsilon \cos(2\pi z/\Lambda)]$ ) with  $w_0 = 4 \mu\text{m}$  and  $\varepsilon = 0.05$ . For this particular design the transition strengths  $|M_{nm}|$  can be obtained as a function of the Bloch wave vector, as shown in Fig. 2. This figure indicates that in this structure inter-band direct transitions are most effective between the second and third band. Forbidden transitions can also be identified at the band edges ( $k = 0, \pm\pi/D$ ). We consider two examples of FB mode Rabi oscillations. The strongest one is between the second and third band at  $k = -\pi/D$ . The beam evolution in the modulated lattice, when the FB mode  $\phi_{-\pi/D,2}$  is excited, is shown in Fig. 3(a). The power distribution between the second and third band is also depicted in Fig. 3(b) as a function of propagation distance. As we can see, complete power exchange from the second to the third band takes place after 1.2 cm of propagation. Before this point, the two involved FB modes interfere, leading to secondary power oscillations. A second example of a weak FB mode transition is that of  $1 \leftrightarrow 3$  at  $k = 0$ . The beam propagation (when the FB mode  $\phi_{0,1}$  is excited), as well as the projected power distribution to the two bands, are given in Fig. 4(a), (b), respectively. The conversion efficiency is lower and the oscillation period longer ( $\sim 6$  cm), since this transition is weaker than the previous one, as expected from the selection rule diagram of Fig. 2. In both cases the agreement between coupled mode theory and BPM simulations is excellent.

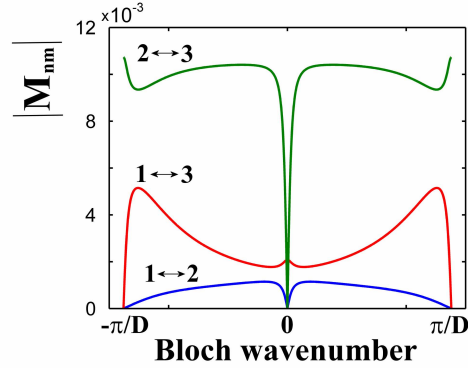


Fig. 2. Selection rule diagram for direct transitions between the first three bands.

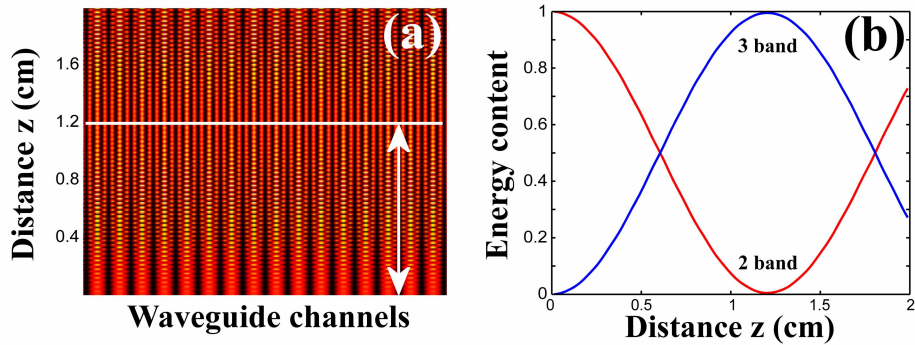


Fig. 3. (a) Intensity evolution pattern of a strong direct transition from the second to the third band at Bloch wave number  $k = -\pi/D$ , and (b) the corresponding total energy in second (red line) and the third (blue line) band, respectively, as a function of propagation distance. The arrow in (a) indicates the location where maximum energy exchange occurs.

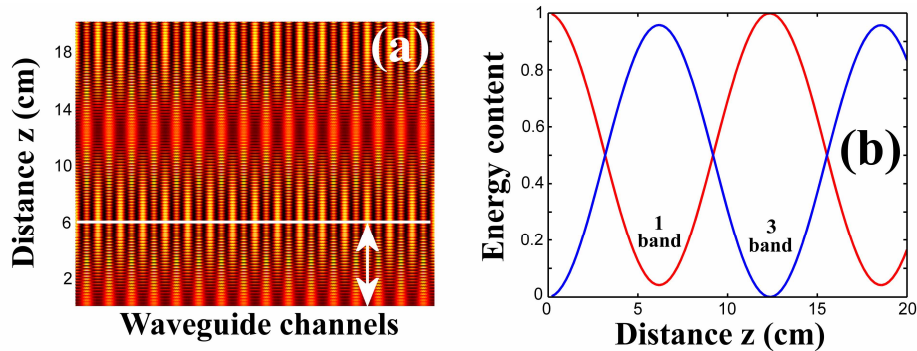


Fig. 4. (a) Intensity evolution pattern of a weak direct transition from the first to the third band at Bloch wave number  $k = 0$ , and (b) the corresponding total energy in the first (red line) and the third (blue line) band, respectively, as a function of propagation distance. The arrow in (a) indicates the location where maximum energy exchange occurs.

#### 4. Nonlinear energy exchange in modulated lattices

Another exciting possibility is to examine whether *nonlinear power exchange from gap to gap* can also occur in such systems. Unlike the cases described above, power exchange between self-trapped states is a direct outcome of nonlinearity. This is because lattice solitons reside in

the gaps of an otherwise defect-free lattice [5, 6, 11-13]. In essence, this may take place while the wave itself maintains its particle nature. In this case, the wave propagation in the  $z$ -modulated periodic potential is governed by the nonlinear Schrödinger equation

$$i \frac{\partial U}{\partial z} + \frac{\partial^2 U}{\partial x^2} + V(x, z)U + |U|^2 U = 0 \quad (3)$$

A numerical simulation predicting such a nonlinear power exchange from the semi-infinite gap to the second gap in a periodically modulated semiconductor waveguide array is examined. In particular, the beam diffracts under linear conditions, as shown in Fig. 5(a). For higher powers and the same excitation, the field self-localizes and forms a lattice soliton with propagation eigenvalue in one of the two gaps, (Fig. 5(b)). Here, the eigenvalue difference of the two solitons involved is taken to be  $2\pi/\Lambda$ . In other words, the process is properly phase-matched. However, such Rabi oscillations between lattice solitons survive only for a few oscillation periods: in every cycle the soliton becomes wider and some power escapes to extended states.

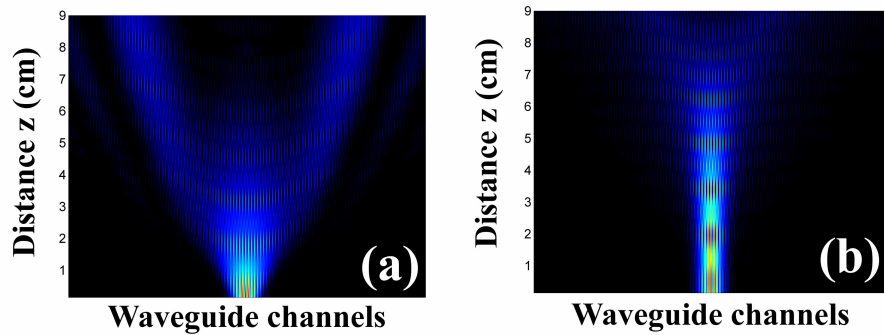


Fig. 5. (a) Diffraction dynamics in a periodically modulated lattice under wide beam excitation, and (b) nonlinear power exchange between a soliton residing at the semi-infinite gap and a soliton in the second gap of the corresponding lattice, under the same excitation conditions.

## 5. Summary

In conclusion, we have demonstrated theoretically that inter-band FB oscillations are possible in waveguide arrays. Such transitions can take place in optical lattices when the channels are periodically modulated along the propagation direction. In the nonlinear domain, oscillations between two different lattice solitons are also possible. Recent experimental observation of such transitions in modulated  $\text{LiNbO}_3$  waveguide arrays [14] is in complete agreement with the presented theoretical results.