One-dimensional bright discrete solitons in media with saturable nonlinearity

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A problem of pulse propagation in a homogeneous nonlinear waveguide array with saturable nonlinearity is studied. The corresponding model equation is the discretized Vinetskii-Kukhtarev equation with neglected influence of diffusion of charge carriers. For periodic boundary conditions, exact homogeneous and oscillating stationary solutions are found. A wide instability region of the homogeneous, array-independent solution is identified. An approximate analytical solution for the bright one-dimensional discrete soliton where the energy is concentrated mainly in a few waveguides is obtained. The soliton stability is investigated both analytically and numerically and a cascade nature of the saturation mechanism is revealed.

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I. INTRODUCTION

A soliton is a type of normal mode of a special class of nonlinear, usually infinite-dimensional mechanical systems often described by integrable partial differential equations such as Korteweg–de Vries, Klein-Gordon, or the nonlinear Schrödinger equation. From a mathematical aspect, solitons are a special nonlinear paradigm since they are associated with Hamiltonians, i.e., conservative systems of a kind which have action-angle variables, so-called integrable (or completely integrable) nonlinear systems. Solitons are stable with respect to collisions with both linear waves and other solitons. However, there are a lot of dynamical systems in nature which could be described by virtue of some nonintegrable partial differential equation. Such systems also have localized, so-called solitary wave solutions that can travel without a change in shape, and a few integrals of motion such as energy, momentum, and number of quanta (power). These solitary wave solutions can exist in bounded media, but often suffer from various kinds of instabilities. In the optical community, it is very common to neglect these differences and just to call these solitonlike pulses simply solitons. Hereafter, we are going to follow this optical notation.

Many different sorts of solitons, which differ in their nonlinear physical mechanism, dimensionality, color, or coherence, have been both theoretically predicted and experimentally observed [1]. But, definitely, photorefractive solitons attract the biggest attention both in the soliton and the optical communication community. Due to the small optical power required for their generation (microwatt level), it is very easy to obtain them experimentally even with continuous-wave lasers and standard optical equipment, and an almost full control of the relevant parameters can be obtained in the experiment. The formation time of these solitons can be as short as milliseconds and below. Moreover, the magnitude of the saturable nonlinearity of photorefractive crystals can be easily driven by adjusting the applied external electrical field.

There are few different classes of photorefractive solitons, but of special interest are steady-state screening solitons that were predicted and observed a few years ago [2,3]. The physical mechanism which lies beyond the generation of screening solitons is rather complicated and therefore we direct the interested reader to some of the articles in which this is explained in detail [4–6]. We shall just mention that this mechanism includes several processes with a retarded temporal response. Both charge separation and the consequent generation of a space-charge electric field under the influence of an external beam require a finite time which is proportional to the dielectric relaxation time. Due to the charge transportation over macroscopic distances via diffusion and drift, this mechanism is also anisotropic and nonlocal.

Bright solitons in saturable bulk media are already well described [7–11]. The biggest differences from solitons in Kerr media are that photorefractive solitons exhibit a stable self-trapping behavior in both transverse dimensions [8] and an inelastic character of collisions between them leading to the fission and fusion of solitons [7,9].

Although many problems in the nonlinear dynamics of spatially extended systems involve continuous media, there are many inherent discrete systems such as Davydov's model for transport of energy in biophysical systems [12], models describing the optical pulse propagation in arrays of coupled optical waveguides [13–16], the model of Scheibe aggregates [17], and others. The major physical parameter in these systems is the interelement distance. The localized states in discrete systems arise through the balance between nonlinearity and linear coupling effects among the adjacent potential wells. The effect of discreteness may significantly change the stability properties of localized states [13,18], collapse

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dynamics [19–21], and other features as compared to continuous systems. Among the nonlinear discrete systems, probably the most interesting (from a practical point of view) are those in nonlinear optics [15,22]. Such discrete optical solitons are found both in arrays of coupled waveguides [13–16] and fibers [22].

The nonlinear waveguide array was introduced in soliton theory 15 years ago [13]. It is suggested that these arrays possess a great potential for various applications such as optical interconnects, beam deflectors, and modulators as well as nonlinear all-optical switches and amplifiers. The first experimental observation of discrete spatial solitons in nonlinear waveguide arrays with Kerr nonlinearity was reported only five years ago [23]. Soon thereafter, waveguides with a negative diffraction were obtained which enable defocusing of light and paved the way to the discovery of the discrete diffraction managed spatial solitons [24]. Discrete gap solitons in modulated (binary) waveguide arrays were predicted [25] and discrete solitons in two-dimensional (2D) networks of nonlinear waveguide arrays have been proposed [26]. Also, the influence of long-range interactions on the nonlinear localized modes in 2D photonic crystal waveguides has been investigated [27]. Optically induced 1D and 2D photonic lattices have been created by virtue of plane-wave interference, and discrete photorefractive solitons in such systems were numerically and experimentally obtained very recently [28].

However, as of yet, there are neither explicit results for the dependence of these localized discrete structures in a saturable media on the system parameters nor information on their linear stability. The objective of our paper is to investigate the existence and stability of 1D bright discrete screening solitons. Our findings could be interesting not only for a particular application in nonlinear optics but also for different discrete biophysics and solid-state physics systems with the same type of nonlinearity. This paper is organized in the following manner: the basic evolution equation is defined in Sec. II, analytical and numerical results concerning homogeneous solution are collected in Sec. III, corresponding results related to the soliton solution are placed in Sec. IV, while the conclusions are given in Sec. V.

II. MODEL EQUATION

The evolution equation of bright 1D optical spatial solitons in bulk photorefractive media, based on the Vinetskii-Kukhtarev model [4] (with the neglected diffusion term), can be written as [6]

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2} - \beta\frac{U}{1+|U|^2} = 0, \qquad (1)$$

where $U=I/I_d$ is a normalized slowly varying envelope of the electric field of the light wave, I is the intensity of the beam while I_d is the so-called dark irradiance, and $\xi=z/kx_0^2$ is a dimensionless coordinate along which the beam propagates. Here x_0 is an arbitrary spatial width and $k=2\pi n_{eo}/\lambda_0$ is the wave number with the unperturbed extraordinary refractive index n_{eo} and light wavelength λ_0. We use $s=x/x_0$ as a normalized transverse coordinate and $\beta=(kx_0 n_{eo})^2 r_{33}E_0/2$ is a positive parameter (r_{33} is the electro-optic coefficient, $E_0 \approx V/L$, where V is a constant bias voltage and L is the width of the photorefractive crystal along the x direction). The crystal is so oriented that its ferroelectric c axis coincides with the x direction. It is also assumed that the incident laser beam is polarized along the c axis (extraordinary refractive index) and that the applied electric field E_0 has a component only in the same direction.

The optical pulse propagation in 1D equidistant nonlinear waveguide arrays with saturable nonlinearity can be modeled, within the nearest-neighbor approximation and with neglected influence of diffusion of charge carriers, by virtue of the following discrete version of the Vinetskii-Kukhtarev equation:

$$i\frac{\partial U_n}{\partial \xi} + \frac{1}{2h^2}(U_{n+1} + U_{n-1} - 2U_n) - \beta\,\frac{U_n}{1+|U_n|^2} = 0, \qquad (2)$$

where U_n is the wave function in the nth nonlinear element $(n=1,\ldots,N)$ with $(U_{N+1}=U_1)$ for the case of periodic boundary conditions, $h=(L-Nd)/Nx_0$ is the normalized distance between two elements, and d represents the width of a single waveguide (usually a few microns). This equation in fact represents a system of linearly coupled nonlinear ordinary differential equations which are not integrable in the general case. It possesses a hidden Hamiltonian structure $i\,\partial U_n/\partial\xi = \delta H/\delta U_n^*$, where H is the Hamiltonian of the system and the asterisk denotes a complex conjugation. Here $H=\Sigma_n[\beta\ln(1+|U_n|^2)+|U_{n-1}-U_n|^2/2h^2]$ is the first conserved quantity of the system, while the second one is a number of quanta (power) $P=\Sigma_n|U_n|^2$. In the small-amplitude limit $|U_n|^2 \ll 1$, this equation passes into the well-known $(1+1)$ discrete nonlinear Schrödinger equation with Kerr nonlinearity. It means that with Eq. (2), under proper conditions, one could also describe various real discrete structures such as a chain of tightened atoms [20,29], the model of dynamics in globular proteins and some molecular crystals [12,30], arrays of Josephson junctions [31], polarons in condensed-matter physics [32], and pulse propagation in short fiber arrays (where dispersion can be neglected) [22].

Hereafter, we have restricted our study to a planar homogeneous array of waveguides in a photorefractive SBN61 ($Sr_{0.61}Ba_{0.39}Nb_2O_6$) crystal, which possesses excellent optical properties and large nonlinear electro-optic coefficients [33]. Permanent waveguides in SBN61 can be fabricated by ion implantation [33]. Usually SBN61 crystals are both a few mm long (z direction) and wide (x direction). The unperturbed extraordinary refractive index for SBN61 is $n_{eo}=2.35$, while the relevant electro-optic coefficient is $r_{33}=280$ pm/V. This crystal is sensitive to both blue and green light. The arbitrary scaling length x_0 is set to 8 μm.

III. HOMOGENEOUS SOLUTION

The stationary solutions are of significant importance because they represent some of the available attractors of the system. Within a linear theory, it shows up that both the exponential perturbations' growth rate and the threshold amplitude for the onset of instability depend on the wave number (and/or on the nonlinear frequency shift) of the stationary solution. There are a lot of reports about the linear stability of these stationary solutions in discrete [13,15,18,21,27,29,34–37], continuous [19,38], as well as in continuous-discrete systems [39,40]. For any set of the system's parameters β and h there exist $3N$ possible stationary solutions such as homogeneous, oscillatory, soliton, and multisoliton solutions. Here we investigate the existence and stability of the simplest homogeneous and soliton solution.

We have found the following exact oscillatory wave solution of Eq. (2):

$$U_n^{\mathrm{osc}}(n,\xi) = \pm\sqrt{\frac{\beta + 2h^{-2}\sin^2(Kh/2) - \nu}{\nu - 2h^{-2}\sin^2(Kh/2)}}\, e^{-i\nu\xi + iKnh}, \quad (3)$$

where K is a discrete wave number and ν is the nonlinear frequency shift. The amplitude of this solution is real if $2h^{-2}\sin^2(Kh/2) < \nu < \beta + 2h^{-2}\sin^2(Kh/2)$ and the corresponding existence region is of width β. It is also possible to find an exact, homogeneous solution

$$U_{\mathrm{hom}} = A_0 e^{-i\nu\xi} = \pm\left(\frac{\beta-\nu}{\nu}\right)^{1/2} e^{-i\nu\xi}. \quad (4)$$

This simple array-independent solution is a special case of the oscillatory one, where $K=0$ is taken. If $0 < \nu \leqslant \beta$, its constant amplitude is real and, opposite to the corresponding solution in Kerr-like media, this one has a limited existence range due to the saturable nature of the nonlinearity.

In order to investigate the linear stability of the homogeneous solution, we follow a standard procedure and introduce a small, complex, array-dependent, in-phase perturbation around this solution $U_n = (A_0 + \delta_n)\exp(-i\nu\xi)$, where $\delta_n(\xi) = [a(\xi) + ib(\xi)]\exp(2i\pi n/N)$ and $(|\delta_n| \ll U_{\mathrm{hom}} = A_0)$. Substitution into Eq. (2), after linearization with respect to the small perturbations and use of Fourier's transform $a, b \propto e^{-i\omega\xi}$, leads to the following dispersion relation:

$$\omega^2 = \frac{4}{h^2}\sin^2\left(\frac{\pi}{N}\right)\left[\frac{1}{h^2}\sin^2\left(\frac{\pi}{N}\right) - \nu\left(1 - \frac{\nu}{\beta}\right)\right]. \quad (5)$$

In the small-amplitude regime, this result coincides (after a simple adjustment of notations and for wave number $q=0$) with the dispersion relation [Eq. (10)] in Ref. [35], where a general approach to modulational instability of discrete nonlinear systems with cubic nonlinearity is described. A sufficient condition for the homogeneous solution to become modulationally unstable is $\omega^2 < 0$ which, together with the fact that $N \approx 100$ and $h \approx 1$, results in the next instability frequency band

$$\frac{\sin^2(\pi/N)}{h^2} < \nu < \beta - \frac{\sin^2(\pi/N)}{h^2}. \quad (6)$$

In the very narrow frequency bands $\Delta\nu = \sin^2(\pi/N)/h^2$ at the ends of the existence interval, this solution is stable with respect to the given form of the perturbation. Note that in these regions, instabilities might occur under some other kind of perturbations. This result is valid in the limit $4\sin^2(\pi/N)/\beta h^2 \to 0$.

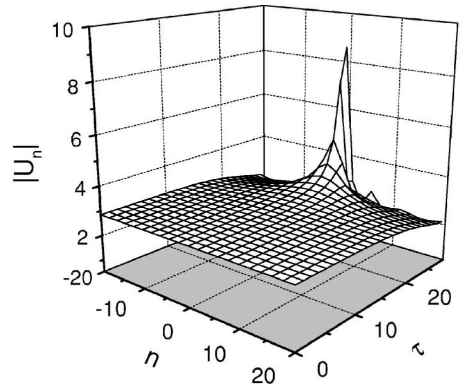

FIG. 1. Example of the time evolution of the electric field across a waveguide array with $N=41$. System's parameters are $\beta=18.2$, $h=0.5$, while $\tau=\xi/2h^2$.

The dispersion relation in Eq. (5) defines the instability growth rate spectra for the frequency band in Eq. (6). In order to confirm our analytical results, we have numerically solved Eq. (2) by a sixth-order Runge-Kutta procedure with regular checking of the conserved quantities P and H. For the initial conditions ($\xi=0$) in the numerical calculations, we have used the perturbed stationary homogeneous solution [Eq. (4)] in the form

$$U_n = U_{\mathrm{hom}}\left[1 + \epsilon\cos\left(2\pi\frac{n - N/2 - 1}{N}\right)\right],$$

where ϵ is a small parameter (here we choose $\varepsilon=0.001$). The growth rates Γ ($\omega = \Omega + i\Gamma$) are numerically estimated from the early stage of the time evolution of the electric field across the waveguide array.

An example of the time evolution of the electric field across a waveguide array with 41 elements and the nonlinear frequency shift $\nu=2$ is shown in Fig. 1, illustrating the development of the modulation instability. The values of the system parameters in this work (except in Fig. 6) are $\beta=18.2$ (we choose the blue light from an argon-ion laser with a wavelength of 488 nm) and $h=0.5$, which correspond to an external applied electrical field $E_0=4.5$ kV/cm and an interelement distance of 4 μm. The numerically estimated (circles) and analytically calculated values (dashed lines) of the growth rates Γ in the central waveguide are plotted over the instability region, which is given by Eq. (6) for $\Omega=0$ and for arrays with $N=5$, 15, and 41 waveguides in Fig. 2. The agreement between the numerical and analytical results is fairly good with only small discrepancies for the array with $N=5$ waveguides in the region of medium values of ν.

This result indicates the presence of exponentially growing modes in the system, giving no predictions about the subsequent nonlinear evolution stage. It is shown that discrete systems with Kerr nonlinearity, instead of a collapse behavior that is observed in the multidimensional continuous case [19], exhibit a quasicollapse behavior leading to the formation of localized structures in the form of discrete solitons [34]. The Kerr nonlinearity is just the small-amplitude limit of the saturable nonlinearity, therefore a similar quasicollapse process and existence of bright 1D discrete screen-

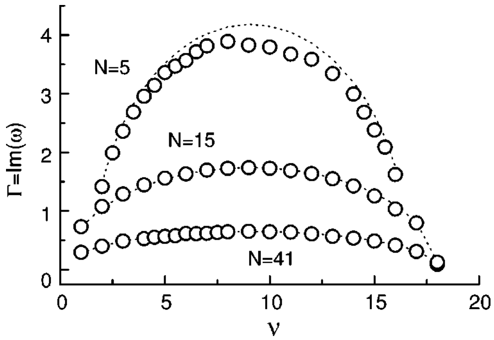

FIG. 2. Comparison of the numerical (circles) and analytical (dashed line) results for the growth rate spectra $\Gamma = \mathrm{Im}(\omega)$ for the nonlinear waveguide arrays with $N=5$, 15, and 41 elements.

ing solitons, where the energy is localized only in between a few central waveguides of the nonlinear array, are fully expected.

IV. SOLITON SOLUTION

As for Kerr-like media [34], it is possible to find an approximate expression for such a narrow discrete photorefractive soliton. Namely, from Eq. (2), in the symmetric case $U_{n-m}=U_{n+m}$, $(m=1,2,\ldots)$ with $|U_n| \gg |U_{n\pm1}| \gg |U_{n\pm2}|$ one can get

$$U_n = \pm \sqrt{\frac{\beta + \dfrac{1}{h^2} - \nu}{\nu - \dfrac{1}{h^2}}}\, e^{-i\nu\xi},$$

$$U_{n\pm m} = \frac{U_n}{\left[2h^2\left(\beta + \dfrac{1}{h^2} - \nu\right)\right]^m} \qquad (7)$$

for the wave functions in a dominant element and its neighbors. The result for the neighbors is valid only under the restriction that $|U_m|^2 \to 0$. Such localized states are possible neither in the small-amplitude regime nor in the oversaturation regime. Figure 3 depicts the intensity distribution along the central part of the nonlinear waveguide array for $\nu=5$, $\beta=18.2$, and for three different values of the waveguide distance h. The symbols denote the intensity in each waveguide, while lines represent just a guide to the eye. In the case $h=1.5$, the waveguides are well separated and there is almost no coupling between them resulting in a discrete soliton with the energy almost completely concentrated in the central waveguide. Decreasing the distance will increase the coupling, thus small satellites in the first neighbors form and the discrete photorefractive soliton starts to spread. By further reducing the distance, more and more elements of the array are excited, thus the localized structure becomes wider. Obviously, in the saturation region, where these solitons become wider, it is not justified to neglect the amplitude of the first neighbors in comparison to the central element's amplitude. By solving Eq. (2) for the case $\nu=h^{-2}$, with the following corrected amplitude ratios $A_0 > A_1 \gg A_2 \to 0$ one can get the next approximate expressions for the pulse amplitudes in the three central elements of the nonlinear array:

$$A_0 = \pm \sqrt{\frac{\beta^2 h^4 - 2 \pm \beta h^2 \sqrt{4 + \beta^2 h^4}}{2}},$$

$$A_1 = \beta h^2 \frac{A_0}{1 + A_0^2}. \qquad (8)$$

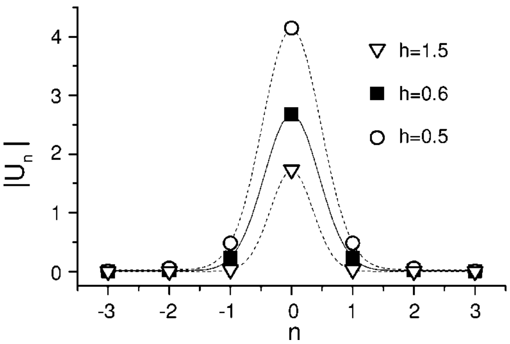

FIG. 3. Intensity patterns of discrete solitons for three different values of the waveguide distance h. The symbols denote the intensity in each waveguide while lines represent a guide to the eye.

We applied a Vakhitov-Kolokolov [38] criterion which gives the answer about the system's stability with respect to small longitudinal perturbations. With an assumption that the system's total power P is shared only between three waveguides, one may obtain

$$P = \frac{1 + 2h^4(\beta + h^{-2} - \nu)^2}{2h^4(\nu - h^{-2})(\beta + h^{-2} - \nu)}. \qquad (9)$$

According to [38,27], this localized structure is stable with respect to the small longitudinal perturbations if the power P is a monotonically decreasing function for any value of the frequency ν. This is both a necessary and sufficient condition for the stability in the discrete system [21]. The analytical form for the power P of discrete screening solitons placed on the center of the lattice as a function of the nonlinear frequency shift ν is presented in Fig. 4(a) (solid line). As the power is a monotonically decreasing function of ν, one might conclude that bright 1D discrete screening solitons are stable with respect to small perturbations. In the small-amplitude regime, this result confirms a conclusion about the stability of the corresponding nonlinear mode (odd, unstaggered, and symmetric) from Ref. [37], where the stability of strongly localized modes was investigated by virtue of a direct linear analysis. In order to study dynamics of the discrete systems in media with saturable nonlinearity, it is necessary to use numerical simulation because Eq. (2) is not integrable in a general case. These numerical results are especially valuable in the deep saturation regime where our approximate theoretical solution fails. The numerically calculated power is given in Fig. 4(a) (dotted line). The agreement with our analytical results is fairly good, except in the big amplitude regime. However, despite a small bend near

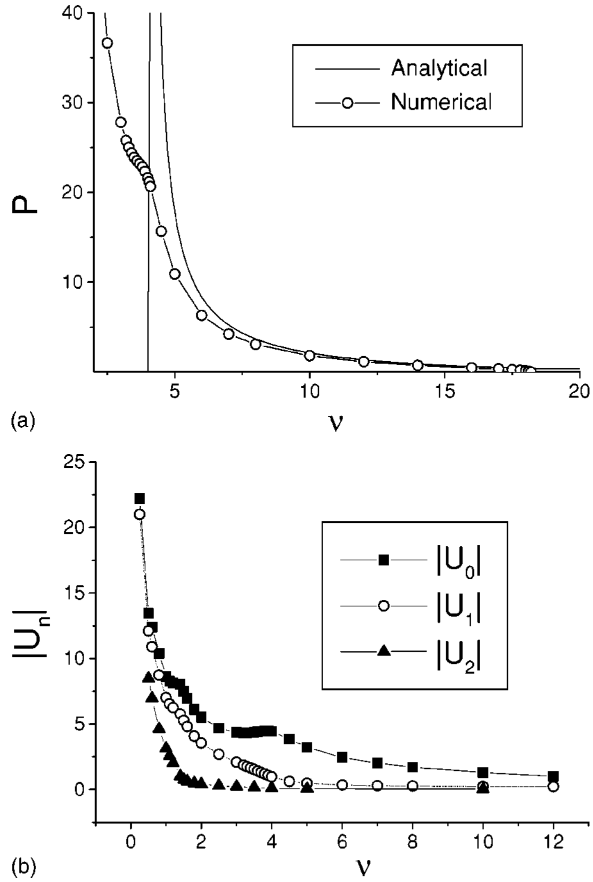

FIG. 4. (a) Power of discrete screening soliton P as a function of nonlinear frequency shift ν ($\beta=18.2, h=0.5$). (b) Amplitude in the central waveguide and its first and second neighbors vs frequency ν for the same parameters as in (a).

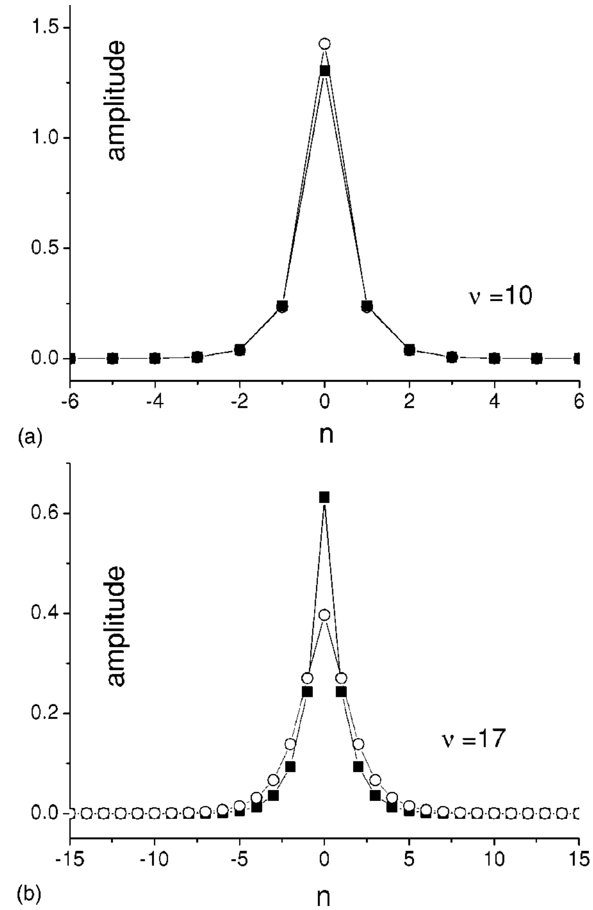

FIG. 5. Amplitude of discrete screening soliton ($\beta=18.2, h=0.5$) vs waveguide number n for (a) $\nu=10$ and (b) $\nu=17$. The numerical results are given by circles while the analytical predictions are given by squares.

the left asymptote ($\nu=h^2$), the curve is still monotonically decreasing, thus confirming the soliton stability.

We also explore the behavior of discrete solitons in the regime of saturation in detail. In Fig. 4(b), where the dependence of the soliton amplitude on the frequency for the central element and its first and second neighbors is presented, one can notice a gradual transition to the saturation regime. The central element goes first into saturation (for $\nu \approx 4$), while the amplitude in its neighbors keeps rising monotonically until $\nu \approx 1.5$, when the first neighbors go in the saturation too. This cascade saturation mechanism can explain the observed bend of the numerical curve for $\nu \approx 4$ in Fig. 4(a). Indeed, the amplitude of the central element of the discrete soliton grows with the increase of the power P until it reaches the saturation level. The further increase of P is an outcome of the growing amplitudes of the first neighbors. It is plausible to expect that the amplitudes of the second neighbors, etc., behave in a similar manner. Thus increasing of P does not lead to a continuous energy localization into a single waveguide and its decoupling from the rest of the array as in the case of the discrete media with cubic nonlinearity. Instead, it leads to a widening of the localized structure. Moreover, by replacing $\beta=18.2$ and $h=0.5$ in Eq. (8) we obtain $A_0=4.545$, $A_1=0.955$ while the corresponding numerical values are $A_0=4.439$, $A_1=0.976$.

Figure 5 depicts a comparison between analytical and numerical results for the possible stationary states of the discrete screening soliton, which is placed at the center of the array consisting of $N=101$ elements. For intermediate frequencies [Fig. 5(a)], narrow discrete solitons are formed. By lowering the soliton amplitude, our numerical simulation suggests a widening of the localized structure [Fig. 5(b)]. It is interesting to mention that for the same initial conditions in the small-amplitude Kerr regime, numerics revealed that both localized and oscillatory solutions are possible. In this region, the slopes of the curves are very small and practically all corresponding soliton solutions are marginally stable.

In Fig. 6, a typical example of a discrete photorefractive soliton propagation along an array with $N=101$ is presented. Our focus is on the central part of the array. Similar patterns are achieved in a wide interval of ν for discrete solitons given by Eq. (7). Note that a qualitatively similar behavior, where the input beam evolves into a stable discrete soliton, is obtained with initially narrow Gaussian, sech, and nearly rectangular pulses (which are more natural in the experiment). We would like to underline that the parameters in Fig. 6 (a waveguide distance of 3 μm and $E_0=9$ kV/cm) are very close to the practically achievable values in the crystal SBN61. For the higher values of ν, the oscillatory solutions are also observed.

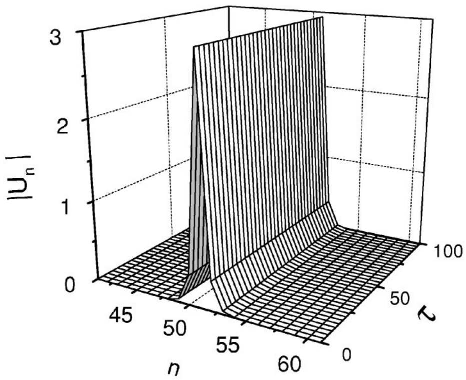

FIG. 6. Propagation of a discrete screening soliton along the array which consists from $N=101$ waveguides ($\beta=36.4$ and $h=0.375$).

V. SUMMARY

In conclusion, the discrete nonlinear evolution equation which describes pulse propagation in a planar homogeneous array of waveguides in media with a saturable nonlinearity is studied. In the case of periodic boundary conditions, two exact (homogeneous and oscillatory) and one approximate (solitonlike) stationary solution are found. A linear stability analysis of the homogeneous solution is performed and analytical expressions for the corresponding instability thresholds and the growth rate spectra are calculated. The growth rate depends on the nonlinear frequency of the solution. Our results show the existence of two cutoff frequencies and a wide instability region in between, where the homogeneous solution is modulationally unstable. In addition, the instability thresholds and the growth rate spectra are numerically calculated for discrete systems with a different number of elements and compared with the corresponding analytical results. Here a very good agreement between numerical and analytical results is found. It is also demonstrated, both analytically and numerically, that such nonlinear waveguide arrays support stable bright one-dimensional discrete spatial solitons. The high amplitude region is studied numerically and a cascade mechanism of saturation in the nonlinear array is found. Finally, the authors would like to emphasize that these explicit analytical and numerical results can be interesting not only for a particular application in nonlinear optics but also for various discrete biophysics and solid-state physics systems with the same saturable nonlinearity.

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