# Photorefractive Spatial Solitons

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**Abstract.** Non-diffracting wave packages or solitons have been the subject of intense study over the last three decades. In particular optical spatial solitons, for which diffraction is exactly balanced by self-focusing in a nonlinear medium, have strongly stimulated the research in the field of solitons in the 90's, especially in photorefractive crystals. Such optical spatial solitons exhibit particle-like behavior in their interactions and stability properties, conserving energy and momentum, and the fascinating results obtained in this field have major consequences in many non-optical systems that can support solitons. This article explains the basic mechanisms that lead to soliton formation, in particular in photorefractive crystals, and gives a short overview of new directions like composite solitons, incoherent solitons formed with spatially incoherent light, and incoherent modulation instability.

## 1 Introduction

When the scottish scientist John S. Russel reported about what he called a 'rounded smooth and well defined heap of water' or 'the great primary wave of translation' in 1834, wave propagation was believed to be a solely linear phenomenon showing broadening or dispersion during propagation. Therefore, it may be understandable that at that time the scientific community had serious problems in believing in what Russel noted eleven years later to be a solitary elevation [1], and it took more than fifty years until the two theoreticians Korteweg and de Vries explained this observation in 1895 [2]. The two dutchmen found out that such a solitary wave must have an unusually large amplitude when compared to the depth of the water, and that in this case the water waves behave and propagate in a completely unusual manner, i.e., they behave as nonlinear waves, or more exact, as waves that propagate in a nonlinear medium. For more then half a century these results gained only little attention, although nonlinear waves where observed in many different wave supporting systems like electron gas in plasmas or phonons in solids. However, it was in 1965 when Zabusky and Kruskal realized that if two of such solitary waves intersect or collide with each other, they may completely maintain their amplitude and shape [3]. Because this behavior is closely related to the collision of particles, they named these nonlinear waves 'solitons'. Following this discovery, a large amount of theoretical and experimental work was done in this new field of nonlinear wave propagation or soliton physics [4–6].

A large amount of todays knowledge on solitons and their behavior upon collision has been obtained by using optical systems, either optical beams (in the spatial domain) [7–10] or optical pulses (in the temporal domain) [11–14] that propagate in a nonlinear optical medium. Such materials possess significant optical nonlinearities, which means that the materials' properties are modified by the light itself. The formation of spatial solitons can be understood as a result of an exact balance between the tendency to broaden because of diffraction and the nonlinear self-focusing. Similarly, temporal solitons form when the natural chromatic dispersion is exactly compensated by the nonlinear self-phase modulation. An intuitive picture for understanding soliton formation is a focused optical beam that gets self-trapped in its own written waveguide. When a narrow light beam travels through a linear medium without affecting the materials' properties, it undergoes natural diffraction and broadens during propagation. The narrower the beam is at the beginning, the larger is its spatial divergence. One of the simplest realization of a nonlinear optical medium is a Kerr-type material where the refractive depends on the light intensity. If the light-induced refractive index change is positive, i.e., the refractive index is increased in the region of higher intensity, a narrow beam is self-focused by the induced nonlinear lens. It is obvious that there must exist a certain strength of the lens where the spatial diffraction of a narrow optical beam is exactly balanced by the self-lensing effect: a bright optical soliton has formed that propagates without diffraction. Dark solitons, by the same definition, are dark stripes or notches on an otherwise homogeneous intensity background, which do not change their profile during propagation, too [10]. In this case, a self-defocusing nonlinearity acting upon the illuminated parts balances the diffraction of the dark notch. A schematic view of this picture of diffraction of bright and dark beams that is balanced by nonlinear focusing and defocusing, respectively, is given in Fig. 1.

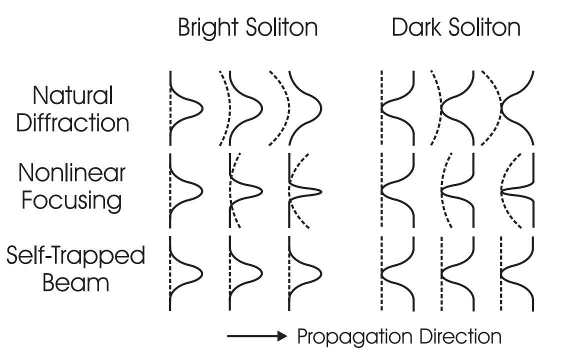


**Fig. 1.** Amplitude (solid lines) and phase fronts (dashed lines) for a bright (left hand side) and dark (right hand side) beam: Natural diffracting beam (upper row), nonlinearly focused beam (middle row) and balance of diffraction and focusing (lower row), leading to soliton propagation

This article is devoted to optical spatial solitons in photorefractive cystals [15,16]. In this class of materials large nonlinear index changes can be obtained at a very small light power level that is in or even below the microwatt regime. Depending on the experimentally controllable parameters, both nearly Kerr-type and saturable nonlinearities can be obtained. In the next section, some basic properties of these two types of nonlinearities will be discussed as well as some fundamentals of photorefractive materials. Section three is devoted to the interaction of solitons, where two possible scenarios, namely coherent and incoherent interactions, are discussed separately. The last section deals with soliton formation and its precursor, modulation instability, using partially spatially incoherent light.

## 2 Spatial Solitons

In a nonlinear optical medium, narrow light beams can propagate without any spatial diffraction, thus forming spatial optical solitons. This happens when an optical beam changes the materials refractive index in such a way that it generates a focusing positive lens. Obviously, this results in an optical waveguide, as now the refractive index in the center of the beam is higher than that at the beam's margins. If the optical beam is also a guided mode of this graded-index waveguide, the reciprocity criterium for spatial soliton formation is fulfilled: a soliton forms when the localized wave-packet induces a potential and gets trapped in it, thus becoming a bound state in its own induced potential.

### 2.1 Kerr-Type Nonlinearity

Kerr nonlinearities are characterized by an instantaneous refractive-index change $\Delta n$ that is proportional to the light intensity $I$, $\Delta n = n_2 \cdot I$, with $n_2$ as the nonlinear coefficient. This nonlinearity is the result of a weak anharmonicity of the elongated electrons of the medium and therefore, at sufficiently high intensities, nearly all media show a noticeable Kerr effect, including crystals, liquids or even gases like air [17]. The governing equation for soliton propagation in Kerr media is the nonlinear Schrödinger equation with a cubic potential [18]. It has been theoretically shown that only one-dimensional (1D) bright solitons, i.e., solitons that are trapped in only one transverse dimension, can propagate stable in a 1D planar medium [5]. Bright 1D Kerr solitons in a two-dimensional (2D) bulk medium suffer from transverse instabilities [6], and 2D Kerr solitons undergo catastrophic collapse [4]. As a consequence, stable 1D Kerr solitons can only be observed in slab waveguides but not in volume samples. In an earlier experiment in 1985, Barthelemy et al. were able to form stable Kerr solitons in liquid $CS_2$ by arresting the transverse instability with an additional interference grating [7]. However, the first true Kerr soliton was observed in a single-mode glass waveguide by Aitchison et al. in 1990 [8].

### 2.2 Saturable Nonlinearity

As early as in 1974 Bjorkholm and Askin were the first to demonstrate spatial soliton formation in a cell filled with sodium vapor [19], and it was about twenty years later in 1993 when Duree et al. [20] demonstrated a stable 2D spatial soliton in another bulk medium, specifically a photorefractive crystal. Both groups used a nonlinearity which has a saturable nature of the form $\Delta n = \Delta n_{\rm sat} \cdot I/(I + I_{\rm sat})$. Here $\Delta n_{\rm sat}$ is the saturated nonlinear refactive index change, $I$ is the intensity of the bright beam, and $I_{\rm sat}$ is the saturation intensity. In the early 1990's it has been theoretically shown that for such a type of a saturable nonlinearity, the catastrophic collapse of 2D solitons in bulk media can be arrested [21]. This is due to the fact that for this type of the nonlinearity the nonlinear index change $\Delta n$ cannot exceed a certain value $\Delta n_{\rm sat}$, and therefore a higher intensity as a result of stronger self-focusing leads to a broader waveguide profile. Because the fundamental mode of such a broader waveguide has also a larger diameter, this can compensate for the stronger self-focusing and leads to a stable 2D soliton in a 2D or bulk medium.

Photorefractive materials are noninstantaneous nonlinear media, where charge carriers are optically excited from impurities and redistributed by different charge transport mechanisms [15,16]. After numerous cycles of excitation and re-trapping which defines the response time of the material, these charges are finally trapped in nonilluminated regions of the sample by deep centers within the bandgap of the material, leading to a space charge field $E_{\rm sc}$ that modulates the refractive index $\Delta n$ via the electrooptic effect, $\Delta n = -0.5 n^3 r E_{\rm sc}$, where $r$ is an electrooptic tensor element. Corresponding to the different mechanisms of charge transport in photorefractive crystals, different types of solitons have been identified in these materials, namely the screening spatial soliton [21,20] that relies on drift of charges in an external field, and the photovoltaic soliton [22,23] that is driven by the bulk photovoltaic effect.

A schematic picture of photorefractive soliton formation is shown in Fig. 2. Let us consider a narrow beam with intensity $I$ that propagates in a photorefractive crystal across which an external electric field $E_0$ has been applied. The direction of this field is in such a way that the refractive index $\Delta n$ is decreased via the electrooptic effect. Due to the photoexcited charge carriers the photoconductivity is increased in the illuminated region of the crystal, and consequently the external field is at least partially screened in the illuminated part of the sample [20]. This leads to a negative dip of the overall electric field $E$, whereas this field is almost not changed in the unilluminated part of the crystal. As can be seen from the minus sign in the definition of the electrooptic effect (provided that $r$ is positive) this results in a positive refractive-index change and finally leads to self-focusing of the beam.

From the experimental point of view, photorefractive crystals enable a relatively simple realization of spatial optical solitons. All parameters influencing the soliton formation and their properties can be easily controlled. For

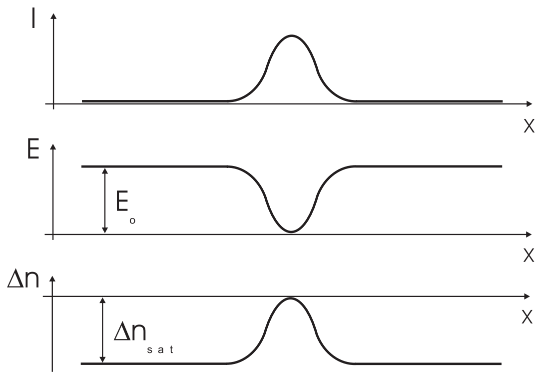


**Fig. 2.** Formation of a bright photorefractive soliton. In the region illuminated by a narrow beam the external electric field is screened, leading to a local positive refractive-index change via the electrooptic effect

example, the maximum size $\Delta n_{\mathrm{sat}}$ of the nonlinearity may be adjusted by applying an appropriate external electric field $E_0$ to the sample, and the degree of saturation (that depends on the conductivity ratio inside and out off the region illuminated by the beam) can be adjusted by illuminating the sample homogeneously with an additional background beam, which provides a homogeneous background conductivity. For most experiments with photorefractice solitons, strontium-barium niobate (SBN) crystals have been used [21,20,24–26]. This material has a large electrooptic tensor element $r_{33} \approx 280\,\mathrm{pm/V}$, which enables soliton formation at rather small values of the externally applied electric field. An example of soliton formation in a planar SBN waveguide is given in the following figure [25].

In Fig. 3(A) the soliton formation starting from the initially divergent HeNe laser beam (632.8 nm) as a function of the externally applied electric field is illustrated. Here the initial beam diameter at the input plane is $d_{\mathrm{in}} =$

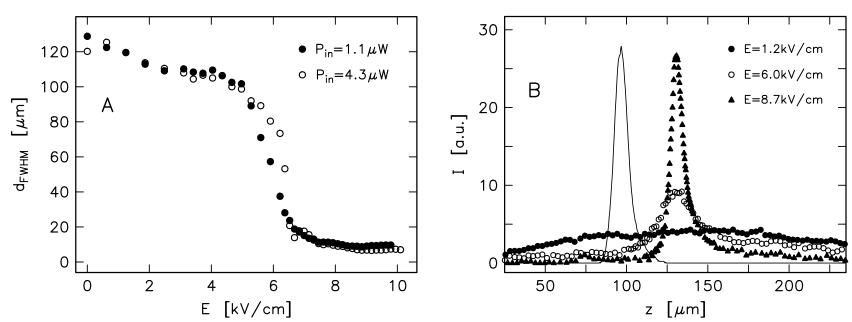


**Fig. 3.** Spatial soliton formation in a planar SBN waveguide. (A) Beam diameter $d$ (FWHM) at the output face of the waveguide as a function of the electric field $E$ and for two different input powers $P_{\mathrm{in}}$ of the HeNe laser. (B) Intensity profiles $I(z)$ on the output face measured for different external electric fields $E$ and for an input power of $P_{\mathrm{in}}$=4.3 $\mu$W. The solid line shows the input beam profile (shifted for better viewing)

$10\,\mu$m, and without electric field it increases because of diffraction to $d = 125\,\mu$m at the output face. For electric fields larger 5 kV/cm the dependence $d(E)$ shows a threshold-like behavior corresponding to the formation of a light-induced waveguide channel that traps the initial light beam. Above this threshold, the beam diameter changes only slightly with electric field. The corresponding intensity profiles $I(z)$ at the exit face of the waveguide are given in Fig. 3(B) for different electric fields $E$.

Up to now only the self-trapping of single or scalar optical beams has been considered, where beam propagation is governed by a single equation. However, there exist also the possibility to trap beams that consist of two or more different components. As now the corresponding equations consist of a whole set with one coupled equation for each component, these creatures are called vector solitons. A prerequisite for vector solitons to exist is that any interference between the different components or modes is absent and does not contribute to the nonlinear refractive-index change. The simplest case has been suggested by Manakov already in 1974 [27] and realized recently by Kang et al. in 1996 [28]. He has shown that temporal vector solitons that consist of two ortogonally polarized modes can exist in optical fibers that exhibit a nonlinear Kerr effect. Other techniques used to form vector solitons in photorefractive crystal is to use mutually incoherent soliton components [29] or beams with slightly different optical frequencies [30].

## 3 Soliton Interactions

Among the most interesting properties of optical solitons is the nonlinear interaction that takes place when two solitons intersect or propagate close enough within the medium so that the evanescent fields of the guided beams at least slightly overlap. These interactions are commonly referred to as collisions. In purely Kerr-type media, it is well known that solitons, in most respects, behave as particle-like objects, and the identity of each soliton is maintained in the interaction [3]. However, solitons in materials with saturable nonlinearity can behave completely different and show a much more rich spectrum of different collision outputs. Both, repulsive and attractive forces between the interacting light beams, can lead to fusion, fission, spiraling, or energy exchange of the solitons [31–33]. The interaction of coherent solitons critically depends on the relative phase of the interacting light beams [31,32]. However, in a soliton-based device it may be difficult to keep this phase relation constant during propagation of the involved beams, and the output of the interaction will be rather difficult to control. To avoid these potential problems, one may consider also the interaction of mutually incoherent solitons, where phase relations do not affect the interaction [34].

### 3.1 Coherent Interactions

When the nonlinear medium can respond to interference effects of the interacting beams, coherent interactions occur. This is the case for instantaneous nonlinearities like the Kerr effect, and for noninstantaneous nonlinearities (including the photorefractive effect) provided that the interference patterns are temporally stable. The latter situation implies that the phase relation of the interacting solitons has to be kept stable for a time that is considerably larger then the mediums response time. Phase-dependent interacting forces between two coherent photorefractive solitons have been observed in different bulk and waveguide materials including BTO [35] and SBN crystals [36], leading to attraction or repelling of colliding beams as well as to energy exchange between them. Here a proper choice of relative phase and intersecting angle has resulted in the fusion of two solitons and the generation of a third beam upon interaction [37]. Very recently, the annihilation of solitons as a result of the interaction of three spatial solitons in a SBN sample has been demonstrated [38].

An example of a coherent soliton collision is given in Fig. 4, where two beams intersect under a small angle $2\Theta$ inside a planar SBN waveguide [33]. Shown is the intensity profile on the endface as a function of the relative phase $\Phi$ of the two solitons. When the two beams are in phase (antiphase), they interfere constructively (destructively) and therefore increase (reduce) the refractive index in the intersection region. An increased refractive index leads to a deflection of both beams towards each other, and for a proper choice of the intersection angle the two beams merge together as can be seen in Fig. 4(A) (solid line). For the antiphase case ($\Phi = 180°$) their distance on the endface increases to $50\,\mu$m (dotted line) when compared to the initial separation of $35\,\mu$m of the beams without nonlinearity. The exchange of energy between two intersecting solitons having a relative phase difference of $\Phi = \pm 90°$ is shown in Fig. 4(B). In both cases, a large part of the intensity initially guided in one beam is coupled into the other one. The direction of energy transfer solely depends on the sign of the relative phase difference of the two beams.

### 3.2 Incoherent Interactions

When the relative phase between interacting light beams changes much faster then the response time of the medium only incoherent interactions can occur [34]. The colliding solitons do not interfere with each other, and the light intensity always increases in the overlap region. When two of these solitons propagate parallel but close to each other (at a distance comparable to the soliton width), or intersect under a small angle that is smaller than the critical angle for guiding in the induced waveguide, their beam trajectories move closer to each other due to the interaction, or eventually fuse together, indicating an attractive force between the beams [34,39,40]. However, for some certain interaction schemes also repulsive forces have been observed [41].

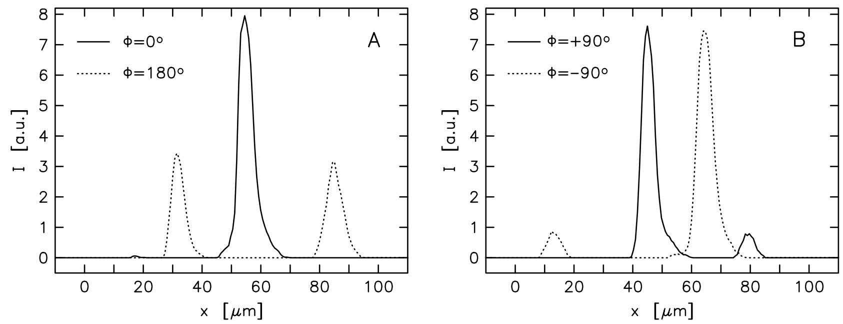


**Fig. 4.** Intensity distribution $I(x)$ on the endface of a planar SBN waveguide for different relative phases $\Phi$ of the two solitons. The two solitons intersect at an angle $2\Theta \approx 0.7°$ and their separation on the endface without interaction is $35\,\mu$m. (A) $\Phi$=0° (solid line) and $\Phi$=180° (dotted line). (B) $\Phi$=+90° (solid line) and $\Phi$=–90° (dotted line), both cases show energy exchange

## 4 Incoherent Solitons

Until recently, solitons were considered to be solely coherent entities. However, incoherent solitons that are formed by partially incoherent light were recently demonstrated experimentally [42,43] and investigated theoretically [44,45]. Incoherent solitons are multimode or speckled beams for which the instantaneous intensity distribution is varying randomly with time. They can exist only in noninstantaneous media, i.e., a nonlinear self-focusing material with a response time that greatly exceeds the characteristic phase fluctuation time of the beam. Such a medium, therefore, responds only to the average intensity of the beam and cannot react to the instantaneous intensity fluctuations. An incoherent soliton forms when the time-averaged intensity induces a multimode waveguide and traps itself in it by populating the guided modes in a self-consistent fashion. Experimentally, the existence of self-trapped spatially incoherent beams was first proved using a rotating diffuser that generated a speckled beam with a correlation distance of only a few micrometers [42]. Later on, also both, spatially and temporally incoherent solitons were formed using white light from a simple incandescent light bulb [43].

### 4.1 Modulation Instability and Pattern Formation

Modulation instability (MI) is a universal process that is inherent to most nonlinear wave systems in nature [46–51]. Because of MI, small amplitude perturbations that originate from noise on top of a homogenous wave front grow rapidly under the combined effects of nonlinearity and diffraction. As a result, a plane wave or broad beam starts to disintegrate during propagation [46–49], which results in spatial filamentation of the wave. Analogous, in the temporal domain a quasi-cw pulse will break into a train of short pulses because of the combined action of self-phase modulation and dispersion [50,51].

It is important to note that MI typically occurs in the same parameter region where spatial or temporal solitons are observed. The relation between MI and solitons is best manifested in the fact that the filaments that emerge from the MI process are actually trains of almost ideal solitons [52,53]. Therefore, MI can be considered to be a precursor to soliton formation.

So far it was always believed that MI is inherently a coherent process and thus it can only appear in nonlinear systems with a perfect degree of spatial and/or temporal coherence. However, recently it has been shown theoretically that MI can also exist in relation with partially-incoherent wave-packets or beams [54]. This in turn leads to several important new features: for example, incoherent MI appears only if the strength of the nonlinearity exceeds a well-defined threshold that depends on the degree of spatial correlation.

Experimentally, incoherent MI has been observed very recently in a biased photorefractive SBN sample illuminated with spatially incoherent light [55]. It has been shown that even in such a nonlinear partially coherent system of weakly-correlated particles patterns can form spontaneously. For the instability to occur, the value of the nonlinearity has to exceed a specific threshold that depends on the coherence properties of the light. Above the threshold, periodic trains of 1D filaments are observed. At higher values of the nonlinearity, these filaments break up into self-ordered arrays or patterns of light spots. Examples of incoherent MI leading to 1D filamentation and 2D pattern formation are given in Fig. 5. Shown is the intensity of the signal beam on the output face of the nonlinear crystal. Fig. 5(A) corresponds to a value of the nonlinearity significantly above the threshold where the filaments have been formed everywhere. When the nonlinearity is further increased, a second threshold is reached: the 1D filaments become unstable and start to break into an ordered array of 2D spots (Fig. 5(B)). It has to be mentioned that in both pictures displayed in this figure, the correlation distance is much shorter than the distance between two adjacent stripes or spots.

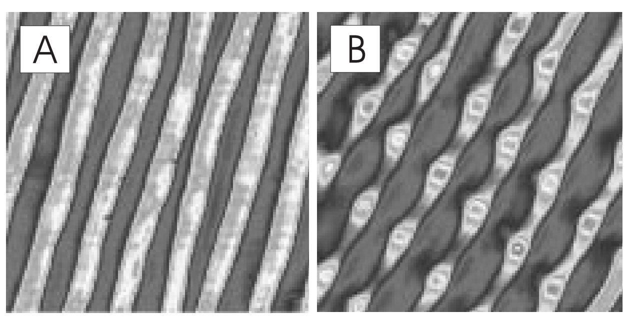


**Fig. 5.** Intensity at the output plane of the crystal. The correlation distance is $l_c$=13 $\mu$m and the displayed area is $0.5\times0.5\,\mathrm{mm}^2$. Plot (A) shows the case above threshold where 1D filaments occur. For much higher nonlinearity the filaments become unstable and form a 2D pattern (B)

The existence of incoherent MI actually reflects on many other nonlinear systems beyond optics: it implies that patterns can form spontaneously from noise in nonlinear many-body systems involving weakly-correlated particles, such as, for example, electrons in semiconductors at the vicinity of the quantum Hall regime, high-$T_c$ superconductors, and atomic gases at temperatures slightly higher than Bose-Einstein-Condensation (BEC) temperatures.

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