Tamm oscillations in semi-infinite nonlinear waveguide arrays

Milutin Stepić, Eugene Smirnov, Christian E. Rüter, and Detlef Kip

Institute of Physics and Physical Technologies, Clausthal University of Technology, 38678 Clausthal-Zellerfeld, Germany

Aleksandra Maluckov

Faculty of Sciences and Mathematics, University of Niš, P.O. Box 224, 18000 Niš, Serbia

Ljupčo Hadžievski

Vinča Institute of Nuclear Sciences, P.O. Box 522, 11001 Belgrade, Serbia

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We demonstrate the existence of nonlinear Tamm oscillations at the interface between a substrate and a one-dimensional waveguide array with either cubic or saturable, self-focusing or self-defocusing nonlinearity. Light is trapped in the vicinity of the array boundary due to the interplay between the repulsive edge potential and Bragg reflection inside the array. In the special case when this potential is linear these oscillations reduce themselves to surface Bloch oscillations. © 2007 Optical Society of America

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The problem of surface waves that could exist at the interface between two different media has been studied for several decades. In linear optics, surface waves in periodic media have been proposed by Kossel.[3] Recently it has been suggested that these surface states may also exist at the interface between a homogeneous medium (substrate) and a nonlinear waveguide array[4] (WA). Up to date, WAs have been fabricated in materials exhibiting cubic,[5,6] quadratic,[7] saturable,[8,9] and nonlocal nonlinearity.[10] Discrete solitons,[5,7,8,11] diffraction management,[6] modulational instability,[9] and Bloch oscillations[12] are just a few examples of phenomena that have been observed in such systems.

The first experimental observation of discrete surface solitons in AlGaAs WAs exhibiting a cubic self-focusing nonlinearity[13] has triggered further investigations of surface waves at the interface between a WA and a substrate. The existence of surface gap solitons in the lattice with cubic self-defocusing nonlinearity has been reported in Ref. 14. A crossover from nonlinear surface states to discrete solitons was studied, too.[15] In these two papers it has been revealed that the vicinity of the edge enables a stable propagation of various localized modes, such as flat-top modes and intersite modes.[16] Very recently, strongly localized surface waves have been observed in WAs exhibiting saturable[17] and quadratic nonlinearity,[18] respectively. In this Letter we reveal the existence of Tamm oscillations at the edge of a semi-infinite WA. These oscillations are the result of an interplay between a repulsive potential that originates from the boundary and the array's periodicity. We calculate this potential for a few different media and reveal that Tamm oscillations are more likely to occur in systems with stronger coupling.

Light propagation in a periodically stratified structure is described by a set of coupled nonlinear ordinary differential equations:

$$i\frac{\mathrm{d}U_n}{\mathrm{d}\xi} + C(U_{n+1} + U_{n-1} - 2U_n) - g(|U_n|^2)U_n = 0, \quad (1)$$

where $g(|U_n|^2)=\beta|U_n|^2$ in cubic (c) media and $g(|U_n|^2)=\beta|U_n|^2/(1+|U_n|^2)$ in saturable (s) media, while the nonlinearity coefficient $\beta<0$ for the self-focusing (f) and $\beta>0$ for the self-defocusing (d) case. Here C is the coupling constant, ξ is the propagation coordinate, and U_n is the normalized electric field envelope in the n-th waveguide. Integrals of motion are power $P=\Sigma_n|U_n|^2$ and Hamiltonian $H_s=\Sigma_n\{C|U_{n-1}-U_n|^2-\beta[\ln(1+|U_n|^2)-|U_n|^2]\}$, $H_c=\Sigma_n[C|U_{n-1}-U_n|^2+\beta|U_n|^4/2]$.

Assuming stationary solutions of staggered form $U_n=F_n\exp[i(-\nu\xi+n\pi)]$ (ν represents soliton frequency) for defocusing cases, and of unstaggered form $U_n=F_n\exp(-i\nu\xi)$ for focusing cases, together with the assumption $|F_0|\gg|F_{\pm1}|\gg|F_{\pm2}|$ for on-site (A) mode and $|F_{\pm1}|\gg|F_{\pm2}|\gg|F_{\pm3}|$ for intersite (B) mode, one may find the following expressions for the maximal amplitude in the array: $F_{0s(f,d)}=\sqrt{(\nu-2C)/(\beta-\nu+2C)}$, $F_{0c(f,d)}=\sqrt{(\nu-2C)/\beta}$, $F_{1s(f)}=\sqrt{(\nu-C)/(\beta-\nu+C)}$, $F_{1s(d)}=\sqrt{(\nu-3C)/(\beta-\nu+3C)}$, $F_{1c(f)}=\sqrt{(\nu-3C)/\beta}$, $F_{1c(d)}=\sqrt{(\nu-C)/\beta}$. In addition, we have $F_{n(>0)}=\alpha^nF_0$ for mode A and $F_{n(>1)}=\alpha^{n-1}F_1$ for mode B. Here $\alpha=\pm C/(\nu-2C)$ for (d) and (f) cases, respectively. Examples of the oscillatory behavior of both on-site and intersite modes are depicted in Fig. 1. Here the energy is too low to overcome the repulsive potential from the edge of the array, so both modes start to move away from the interface until they are Bragg reflected. As this traversing is usually accompanied with radiation, reflected modes lose power and eventually do not reach back to the first channel of the array. As a result, successive oscillations have increasing period lengths and modes

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gradually run away from the edge, eventually being trapped at one channel after several periods.

We calculate the effective repulsive potential as the difference between the truncated potential and a reference potential with periodic boundary condition (i.e., for an infinite array), and present results for mode A in an array consisting of $2n+1$ elements, and mode B in an array consisting of $2n$ elements, where n is an integer. As the peak of these localized structures (with highest amplitude $F\equiv F_0$ for A and $F\equiv F_1$ for B) approaches the edge, more terms of the corresponding Hamiltonian have to be truncated.[15] For example, if the peak of mode A resides in the 0th channel $(-n,-n+1,..,-1,0,1,..,n-1,n)$ and the peak of mode B between the channels −1 and 1 $(-n,-n+1,..,-1,1,..,n-1,n)$, we obtain

$$V_{rep}^{s(p)} = \beta\sum_{i=0}^{p-1}[\ln(1+\alpha^{2(n-a-i)}F^2) - F^2\alpha^{2(n-a-i)}] + F^2C\left[\alpha^{2(n-a-p)} - (1+\alpha)^2\sum_{i=1}^{p}\alpha^{2(n-a-i)}\right], \quad (2)$$

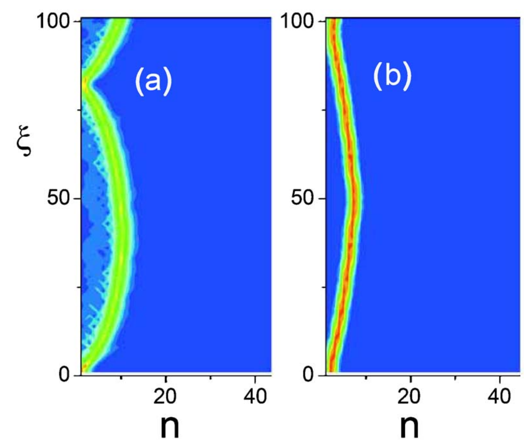

Fig. 1. (Color online) Tamm oscillations of (a) mode A with $C=0.5$, $|\beta|=3.34$, and soliton frequency $\nu=2.3$ launched into the first channel, and (b) mode B with $C=0.5$, $|\beta|=3.34$, and soliton frequency $\nu=4.83$ launched into the second and third channels.

$$V_{rep}^{c(p)} = -\frac{\beta}{2}F^4\sum_{i=0}^{p-1}\alpha^{4(n-a-i)} + CF^2\alpha^{2(n-p-a)} - CF^2(1+\alpha)^2\sum_{i=1}^{p}\alpha^{2(n-i-a)} \quad (3)$$

for lattices with saturable and cubic nonlinearity, respectively. Here p denotes the number of the truncated channels, where $p_{max}=n$, $a=0$ for mode A, and $p_{max}=n-1$, $a=1$ for mode B.

The dependence of the repulsive potential on the distance from the edge is presented in Fig. 2. Stronger coupling results in stronger repulsion [Fig. 2(a)], while stronger nonlinearity decreases the repulsive potential [Fig. 2(b)]. Beams that are strongly pushed off from the edge will experience Bragg reflection inside the array earlier than weakly rejected beams. Thus the same input beam will have shorter spatial periods of Tamm oscillations in arrays with stronger coupling and weaker nonlinearity. The value of the corresponding repulsive potentials also depends on the soliton frequency ν. For a fixed value of ν and fixed values of parameters C and β, the following relation is usually fulfilled: $V_{rep}^{sd} > V_{rep}^{cd} > V_{rep}^{sf} > V_{rep}^{cf}$, but interior elements may permute their position as well. This relation may explain why this effect has not been observed in recent studies in AlGaAs WAs with cubic self-focusing nonlinearity.[4,13] The impact of saturation on the repulsion from the interface in an unstaggered case was discussed recently.[19] Interestingly, in the case when V_{rep} is a linear function from the interface distance the input beam will experience Bloch oscillations, which has considerable potential for application in all-optical devices.[20]

To check our findings we numerically solve the following nonlinear paraxial wave equation [from which, in fact, one can by appropriate discretization obtain Eq. (1)]:

$$i\frac{\partial E}{\partial y} + \frac{1}{2k}\frac{\partial^2 E}{\partial z^2} + k\frac{n(z)+\Delta n_{nl}}{n_s}E = 0. \quad (4)$$

Here light propagation is along the y-axis, E is electric field amplitude, $k=2\pi n_s/\lambda$ is the wavenumber for wavelength λ, and n_s is the substrate index. The periodic index modulation of the WA is $n(z)$, while Δn_{nl} is the nonlinear refractive index change ($\Delta n_{nl} \ll n_s$)

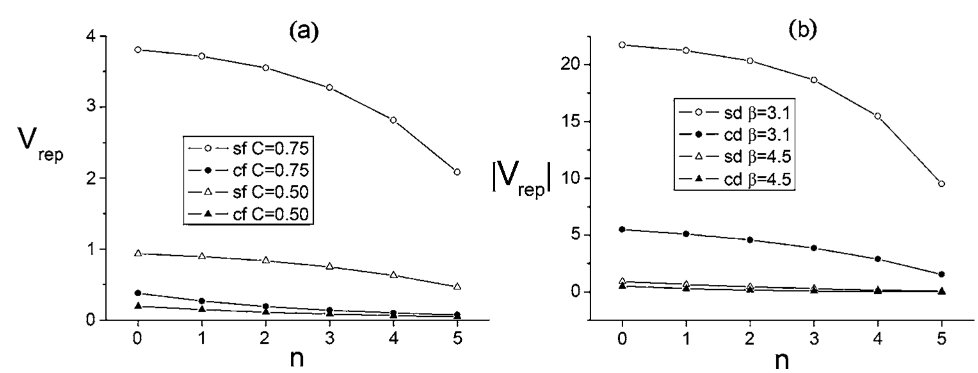

Fig. 2. Dependence of the effective repulsive potential on the distance from the boundary. (a) Self-focusing case: $|\beta|=3.1$, $|F_0|^2=0.24$, and different values of C. (b) Self-defocusing case: $C=0.5$, $|F_0|^2=0.172$, and different values of β.

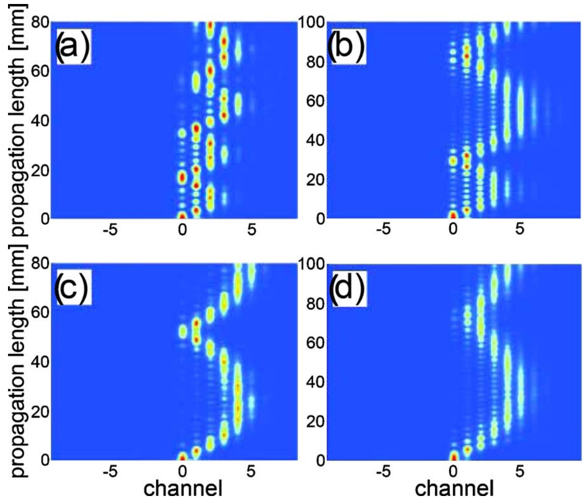

Fig. 3. (Color online) Tamm oscillations in a WA with period 8.4 μm and $\Delta n_0 = 3.7\times10^{-4}$. Input patterns with amplitude ratios 1:0.5:0.1 are launched into the first channel. (a) Self-defocusing saturable case $\Delta n_{nl} = 6.2\times10^{-4}$, $r = I/I_d = 6$; (b) self-defocusing cubic case $\Delta n_{nl} = 4.42\times10^{-4}$; (c) self-focusing saturable case $\Delta n_{nl} = 3.34\times10^{-4}$, $r = 6$; and (d) self-focusing cubic case $\Delta n_{nl} = 2.62\times10^{-4}$.

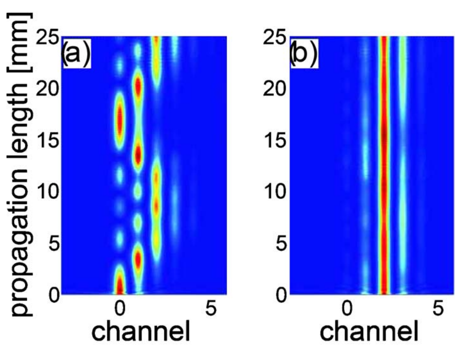

Fig. 4. (Color online) Saturable defocusing WA with input profile and lattice parameters from Fig. 3, but $\Delta n_{nl} = 6.21\times10^{-4}$ and $r = 6$. Input beam is launched into (a) first and (b) third channels.

for cubic ($\Delta n_{nl} = \Delta n_0 I$) and saturable [$\Delta n_{nl} = \Delta n_0 I/(I + I_d)$] media. Here I is the peak light intensity and I_d is the so-called dark irradiance.[8] We used parameters of lithium niobate WA exhibiting a self-defocusing saturable nonlinearity ($\Delta n_0 = 3.7\times10^{-4}$, $\Delta n_{nl} \le 0.001$) and $\lambda = 532$ nm. For such samples the periodically modulated refractive index can be well approximated by a $\cos^2$ function.[9] Some numerical results that support our former findings are given in Fig. 3. Similar results are obtained for different perturbed wider input structures with and without an initial phase offset. Figure 4 may be understood as a numerical proof that these oscillations are indeed a surface effect. Keeping the light intensity necessary to form Tamm oscillations [Fig. 4(a)] constant and shifting the position of input beam towards the interior of the array, the period of Tamm oscillations increases whereby light distribution narrows. In Fig. 4(b) a narrow breather is formed in a channel that is only two channels away from the substrate array interface.

In conclusion, we demonstrate the existence of Tamm oscillations at the interface between a substrate and a one-dimensional homogeneous nonlinear WA. Light is trapped in the vicinity of the edge of the array due to the interplay between the edge repulsion and Bragg reflection. Approximate analytical expressions for the repulsive truncated potential are given for different types of nonlinearities as well as for different system parameters. These oscillations reduce to the case of Bloch oscillations when the repulsive potential is a linearly decreasing function of the distance from the edge of the semi-infinite WA.

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